# Math Power

**November 11, 2020**
☎: 301-251-7014
site: http://www.MathEnglish.com

**By Dr. Li**
E-mail : DL@MathEnglish.com

Name: (First)_____(Last)_____

School: _____ Grade:_____





SAT

# S A T Math Sample

## *Warmups*

1. If $2m = \frac{1}{3}n$ and $n = 11$, what is the value of $\frac{1}{2}m$?

2. Find the roots of $2x^2 - 4 = 5x$.

3. Mr. Workinghard earned $1,100 for 50 hours of work last week. He received 50% more than regular hourly rate for all hours over 40 hours in a week. How much will he get paid if he works 60 hours next week?

4. Given that $(x - 1)^2 + (y + 2)^2 = 0$. Find the value of $x^2 + y^2$.

5. If $x^{44} = 10$, what is the value of $x^{66}$?

6. The first term of a sequence is -3 and every term after the first is 5 more than the term immediately preceding it. What is the value of the 101st term?

7. Which of the following is equal to $(100 + n)/25$?
   A) $(4 + n)/25$
   B) $(20 + n)/25$
   C) $4n$
   D) $4 + n$
   E) $4 + (n/25)$

8. If $a = \frac{1}{2}$, $b = \frac{2}{3}$, and $c = \frac{3}{4}$, what is the value of $(2a + 3b) \div c$?

9. What percent of $x$ is $y$?

10. List I: 2, 4, 7
    List II: 3, 4, 6
    One number is to be selected from each of the list above. What is the probability that both of the numbers selected will be less than 5?

*Question set* **[11 - 13]**

A box contains 3 black pieces of cloth, 2 striped pieces, and 4 dotted pieces. A piece is selected randomly and then placed back in the box. A second piece is then selected randomly.



SAT

11. What is the probability that both pieces are dotted?

12. What is the probability that the first piece is black and the second piece is dotted?

13. What is the probability that one piece is black and one piece is dotted?

14. Tina and Jose are drawing a ticket each from a box containing 3 concert tickets and 5 movie tickets. What is the probability that both will draw concert tickets?

15. If $a$ is a number that is randomly selected from set A = {0, 1, -3, 6, -8} and $b$ is a number randomly selected from B = {-1, 2, -4, 7}, what is the probability that $ab < 0$?

16. Suppose that Jack bought a putter on sale for $39.00 at a discount of 35%. How much discount did he get?

17. Last year, Paul's weight was 150 lb. He gains 30% in weight this year. How much does he weigh this year?

18. Suppose that Lou invested a certain amount of money at 9% interest and $300 more than that amount at 11%. His total yearly interest was $101. How much did he invest at 11%?

19. In Europe tire pressure is measured in kilograms per square centimeter. If 32 pounds per square inch corresponds to 2.24 kilograms per square centimeter, how many kilograms per square centimeter is equivalent to 24 pounds per square inch?

20. Kirk starts jogging at 5 miles per hour. Half hour later Nancy starts jogging on the same route at 7 miles per hour. How long will it take Nancy to catch up with Kirk?

## *Algebra 2*

*Question set* [**21 - 23**]

Basic algebra.

21. When 4 times the number $x$ is added to $12y$, the result is 22. What number results when 10 times $x$ is added to $30y$?

22. When 4 times the number $x$ is added to 12, the result is 8. What number results when 2 times $x$ is added to 7?

23. When 4 times the number $x$ is added to 12, the result is 22. What number results when 10 times $x$ is added to 70?

24. A group of students at Omega High School are using staples and popsicle sticks to build a scale model of the Great Wall of China as part of a project detailing China's history. The number of staples the students will need is three times the number of glue sticks they will need. If the students determine they need a total of 84 stables and glue sticks for this particular project, how many glue sticks will they need?

25. Given a system of equations:
$\begin{cases} \frac{3}{4}x+\frac{5}{6}y=12 \\ 3x-5y=10 \end{cases}$
What is the value of $9x + 10y$?

26. If $4x + 6y = 24$, what is the value of $6x+ 9y-12$?
A) 18 B) 20
C) 22 D) 24

27. What is the value of $h$ in the equation below?
$$\frac{4h-(21-8h)}{3}=\frac{15+6(h-1)}{2}$$

28. Solve $x$ in terms of $k$ the equation:
$$\frac{2x-1}{3}=k$$

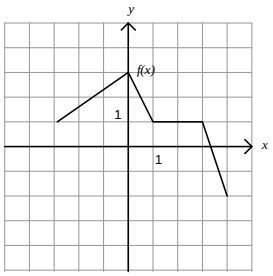
29. Solve $x$ in terms of $k$ the equation:
$3x + 2 = 3(k - x)$

30. Solve $x$ in terms of $k$ the equation:
$2kx - 3 = 3x + 2k$

31. Solve $x$ in terms of $k$ the equation:
$2k(x - 3) = 3(x + 2k)$



SAT

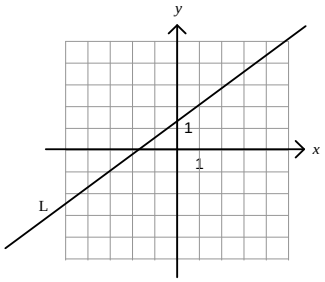



32. The complete graph of the function $f$ is shown in the $xy$-plane above. Which of the following are equal to 1?

I. $f(-3)$
II. $f(\frac{3}{2})$
III. $f(3)$

A) III only B) I and II only
C) II and III only D) I, II, and III

*Question set* [**33 - 34**]




In the figure above, the graph of $y = f(x)$ is shown.

33. Which of the following could be the equation of $f(x)$?

A) $y-1=\frac{3}{4}(x-2)$ B) $y-2=\frac{4}{3}x-1$
C) $y-2=\frac{3}{4}(x-1)$ D) $y-2=\frac{4}{3}(x-1)$

34. Which of the following equation describes the perpendicular line M (not shown) to L, passing through (2,−1)?

A) $3x + 4y = 2$ B) $4x + 3y = 5$
C) $3x - 4y = 10$ D) $4x - 3y = 11$

*Question set* [**35 - 38**]

| $x$ | $f(x)$ |
|---|---|
| 0 | 12 |
| 2 | 0 |
| 4 | 4 |
| 5 | ? |

A quadratic function $f(x)=ax^2+bx+c$, where $a$, $b$, and $c$ are all integers. Some values of $x$ and $f(x)$ are shown in the table above.

35. Which of the following must be a factor of $f(x)$?

A) $x - 2$ B) $x + 2$
C) $x - 4$ D) $x - 5$

36. What is the value of $c$ in the function $f$?

A) 0 B) 2
C) 4 D) 12

37. Let $f(x) = (x - 2)(ax - \beta)$. Find the values of $a$ and $\beta$ by using the given function table.

A) $a$=2, $\beta$=-6 B) $a$=-2, $\beta$=6
C) $a$=-2, $\beta$=-6 D) $a$=2, $\beta$=6

38. Find the value for $f(5)$.
A) 8 B) 10
C) 12 D) -16

39. $-3(2x + 1)(4x - 1)$
Which of the following is equivalent to the expression above?
A) $-45x$
B) $24x^2 + 3$
C) $-24x^2 - 6x + 3$
D) $24x^2 + 6x - 3$

40. If $A = x^2+4x+9$ and $B = x^3+6x-2$, what is $3A + B$?
A) $4x^2+18x+25$
B) $x^3+18x+25$
C) $x^3+3x^2+18x+25$
D) $x^3+3x^2+30x+29$

## *Warm-ups*

41. Barbara is 4 years younger than Kathy. In 3 years, Barbara's age will be $\frac{2}{3}$ of Kathy's age. How old is Barbara now?

42. Find a possible value of $k$ ($\neq 0$) so that the graph of
$f(x)=kx^2- 4x + k$
tangent to the $x$-axis.

43. If $x = 9a^2$ and $a < 0$, express the value of $\sqrt{x}$ using $a$.

*Question set* [**44 - 45**]

Let $k*j$ be defined as the sum of all integers between $k$ and $j$. For example, 5*9 = 6 + 7 + 8 = 21.

44. 7*10 =

45. What is the value of (80*110) - (81*109)?

46. Given $7x - 5y = 13$ and $2x - 7y = 26$. Find the value of $5x + 2y$.

47. $\frac{3x}{x+2} + 1 = \frac{35}{2(3x+1)}$



SAT

48. Simplify the following expression in fraction:
$-6x^3y^2 \div (-4x^2y^6)$

49. Simplify the following expression:
$(-2a^3)(5ab^2) \div (-3a^4b) =$

50. Given $3r - 2s = 0$. Evaluate $\frac{9r^2}{s^2}$.

51. A dinner special at a restaurant consists of soup or salad, a main dish, dessert and coffee or tea. If there are 8 different main dishes and 5 different desserts to choose from, how many different dinner specials can be ordered?

52. In a recent civil service examination, $\frac{1}{8}$ of the candidates failed the first part of the test. Of those eligible to take the second part of the competitive examination, $\frac{5}{7}$ failed. What part of the original candidates were successful in the examination?

53. The surface area of a cube is 150 $in^2$. What is the volume of the cube?

54. If $x < y < 0$, then which of the following must have a value greater than 1?
A) $xy$
B) $x - y$
C) $\frac{y}{x}$
D) $y-x$
E) $\frac{x}{y}$

55. A die and a coin are thrown together. What is the chance of getting a '3' and a 'head'?

56. If \$3000 is invested at 9% interest, how much money must be invested at 12%, so that both will yield the same return?

57. After a discount of 30%, a purse was sold for \$21.63. What was the original price?

58. Phoebe paid $32 for a pair of gold shoes that were listed for $40. What rate of discount did she receive?

59. Two city buses leave from the same station in opposite directions. One travels at 35 miles per hour, and the other travels at 45 miles per hour. In how many hours will they be 60 miles apart?

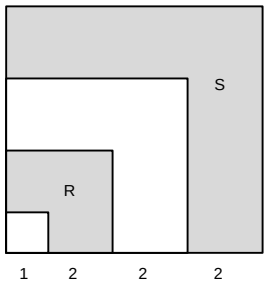
60. Michael and Barry can complete a job in 2 hours when working together. If Michael requires 6 hours to do the job alone, how many hours does Barry need to do the job alone?

### *New Challenges*

61. Find the maximum number of intersection points of N circles.
A) N(N – 3)
B) N(N – 1)
C) N(N-1)(N-2)
D) 2N(N – 1)

62. For how many values of $x$ between 0 and $2\pi$ does $\sin 3x = \frac{1}{2}$?
A) Two
B) Three
C) Four
D) Six

63. Four squares share one common vertex. Each side of the smallest square is 1 in, and the next larger one is 2 in longer, etc. Find the area ratio between the shaded regions R : S.




*Question set* [**64 - 65**]

Factor the following polynomials.

64. $x^4 - 13x^2 + 36$

65. $.01a^2 + .03ab - .54b^2$

66. If $i = \sqrt{-1}$, which of the following is not equal to $i^3 + i$?
A) $(2i)^2 + 4$
B) $2 - 2i^{16}$
C) $2i^{22} - 2$
D) $i^8 - i^4$

*Question set* [**67 - 68**]

Quadratic equations in square root.

67. Solve $\sqrt{21-4x} = 2x - 3$.
A) {-1, 3}
B) {-1}
C) {3}
D) {}

68. Solve $\sqrt{48x-71} = 1 - 3x$. The solution set is
A) {2, 4}
B) {2}
C) {4}
D) {}

*Question set* [**69 - 70**]

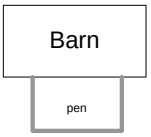
Operations complex numbers. Let
$z_1 = 7 - 24i$
$z_2 = 3 + 4i$

69. $(z_1 + z_2)^2 = a + bi$, where $a, b \in R$. Find $(a, b)$.

70. $\frac{z_1}{z_2} = c + di$, where $c, d \in R$. Find $c^2 + d^2$.

## *Real Challenge*

71. A total length of 36 ft fence is used to make a rectangular pen along the wall of a barn. What is the largest area of pen he can enclose?
(A) 162 ft$^2$ (B) 152 ft$^2$ (C) 142 ft$^2$
(D) 132 ft$^2$ (E) 10 ft$^2$




72. If $r$ and $h$ are positive integers and $\frac{1}{2}r + 21 = 3h^2$, what is the least value of $r$?
(A) 10 (B) 11 (C) 12
(D) 13 (E) 15

73. If $f(a, b)$ is defined as $(a^b) \div (b^a)$. Which of the following attains the largest value?
(A) $f(-2, 3)$ (B) $f(2, -3)$ (C) $f(-2, -3)$
(D) $f(2, 3)$ (E) $f(3, 2)$

74. Laneitre is trying to figure out the heights of 3 people. Here are the facts she knows.

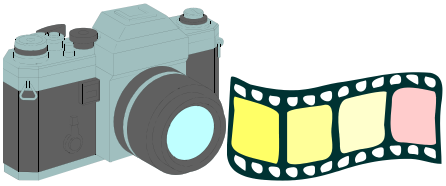
• The sum of the heights of these 3 people is 17 feet 5 inches.
• The shortest person is 5 feet 4 inches tall.
• The other 2 people differ in height by 3 inches.

How tall is the tallest person?
(A) 5 feet 4 inches
(B) 5 feet 11 inches
(C) 6 feet 2 inches
(D) 12 feet 1 inch

75. In the rectangular coordinate plane, what is the distance between the points (2, 6) and (10, -9)?
(A) 15 (B) 17 (C) 19
(D) 21 (E) 25

*Question set* [**76 - 77**]

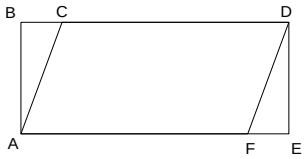
Yolanda shot 720 pictures this year. She used the same number of rolls of 36-shot film as she did of 24-shot film.



76. How many rolls in total did she need if she uses the same number of rolls from both 24-shot and 36-shot film?
(A) 24 (B) 22 (C) 20
(D) 18 (E) 16

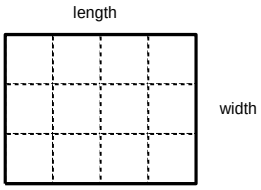
77. How many rolls in total did she need if she uses the same number of shots from both 24-shot and 36-shot film?
(A) 20 (B) 25 (C) 28 (D) 30 (E) 32

78. In the figure below, ABDE is a rectangle.




The length of BD is 15, the length of CD is 10, and the length of AC is 13. What is the area of parallelogram ACDF?
(A) 24 (B) 30 (C) 50 (D) 60 (E) 120

79. The width of a certain rectangle is three-fourths of its length and its area is 108 square meters. Find the width of the rectangle.
(A) 12 m (B) 10 m
(C) 9 m (D) 7 m (E) 5 m




80. In a class of 25 students, the first 10 received test scores averaging 92, while the second 10 received score averaging 84. If the average for the entire class was 86, what was the average score for the last 5 students?
(A) 70 (B) 75 (C) 78 (D) 80 (E) 82

# Answer Key

1. $\frac{11}{12}$

2. $\frac{5\pm\sqrt{57}}{4}$

3. Let $p$ be his regular hourly rate, so $1.5p$ be his overtime hourly rate.
   $40p + 10(1.5p) = 55p = 1100$
   $p = 20$
   $40p + 20(1.5p) = 70p =$ \$1400

4. $x = 1$ and $y = -2$
   $x^2 + y^2 =$ 5

5. $x^{66} = (x^{44})^{\frac{3}{2}} = (10)^{\frac{3}{2}} = 10\sqrt{10}$

6. D
   In general, the $i$-th term of this sequence is $a_i = -3 + (i-1)\times 5$, thus the 101st term is $-3 + (101 - 1)\times 5 =$ 497.

7. E

8. $2a + 3b = 1 + 2 = 3$
   $3\div\frac{3}{4} =$ 4

9. $\frac{100y}{x}$

10. $\frac{2}{3}\times\frac{2}{3} = \frac{4}{9}$

11. $\frac{4}{9}\times\frac{4}{9} = \frac{16}{81}$

12. $\frac{3}{9}\times\frac{4}{9} = \frac{4}{27}$

13. $\frac{4}{9}\times\frac{3}{9} + \frac{3}{9}\times\frac{4}{9} = \frac{8}{27}$

14. $\frac{3}{8}\times\frac{2}{7} = \frac{3}{28}$

15. case I: ($a>0$, $b<0$), #I $= 2\times2=4$
    case II: ($a<0$, $b>0$), #II $= 2\times2 = 4$
    $P$(I or II) $= \frac{8}{20} =$ 0.4

16. $\frac{35}{65} = \frac{7}{13} = \frac{x}{39}$
    $x =$ 21

17. $1 + 30\% = 1.3$
    $150\times1.3 =$ 195 lb

18. Let $x$ be the amount invested at 9%, hence $x + 300$ be the amount invested at 11%. Then, we have
    $9\%(x) + 11\%(x + 300) = 101$
    $9x + 11(x + 300) = 10100$
    $20x = 6800$
    $x = 340$
    $x + 300 =$ \$640 (11%)

19. $\frac{24}{32}\times 2.24 = \frac{3}{4}\times2.24 = 3\times0.56 = 1.68$

20. $\frac{1}{2}\times5 = 2.5$ miles (Kirk was ahead.)
    $7 - 5 = 2$ (Nancy was 2 mph faster)
    $2.5\div2 = 1.25$ (hr) = 1 hr and 15 min

21. $4x + 12y = 22$
    $2x + 6y = 11$
    $10x + 30y = 55$

22. $4x + 12 = 8$
    $2x + 6 = 4$
    $2x + 7 = 5$

23. Don't solve it, but use it.
    $4x + 12 = 22$
    $2x + 6 = 11$
    $10x + 30 = 55$
    $10x + 70 = 95$

24. $3x + x = 84$
    $4x = 84$
    $x = 21$

25. $12\times$①
    $9x + 10y = 144$

26. D
    $4x + 6y = 24$
    $2x + 3y = 12$
    $6x + 9y = 36$
    $6x + 9y - 12 = 24$

27. Multiply 6 to both sides:
    $\frac{12h-21}{3} = \frac{6h+9}{2}$
    $24h - 42 = 18h + 27$
    $6h = 69$
    $h = 11.5$

28. $2x - 1 = 3k$
    $x = \frac{1}{2}(3k + 1)$

29. $6x = 3k - 2$
    $x = \frac{1}{2}k - \frac{1}{3}$

30. $(2k - 3)x = 2k + 3$
    $x = \frac{2k+3}{2k-3}$

31. $(2k - 3)x = 12k$
    $x = \frac{12k}{2k-3}$

32. D

33. C

34. B

35. A
    $f(2) = 0 \Rightarrow x-2$ must be a factor

36. D
    $f(0) = 12$ (use the first entry)

37. D
    $f(x) = (x - 2)(ax - \beta)$



SAT

$f(0) = (-2)(-\beta) = 12 \Rightarrow \beta = 6$
$f(x) = (x - 2)(ax - 6)$
$f(4) = (2)(4a - 6) = 4$
$2a - 3 = 1 \Rightarrow a = 2$

38. C
$f(x) = (x - 2)(2x - 6) = 2(x - 2)(x - 3)$
$f(5) = 2(3)(2) = 12$

39. C

40. C

41. Let $x$ = Barbara's age,
$x+4$ = Kathy's age.
$x + 3 = \frac{2}{3}(x+7)$
$x =$ 5 yrs old

42. $kx^2 - 4x + k = k(x^2 - \frac{4}{k}x + 1)$
To be a complete square, $k = \pm 2$.

43. $-3a$

44. 8 + 9 = 17

45. Let $T_1 = \{x \in Z \mid 80<x<110\}$.
Let $T_2 = \{x \in Z \mid 81<x<109\}$. The set $T_1$ has 2 more elements than $T_2$, 81 and 109. Therefore, the difference of sums of elements in $T_1$ and $T_2$ is 81+109 = 190.

46. Subtract the second equation from the first one.
$5x + 2y =$ -13

47. $\frac{3x}{x+2} + 1 = \frac{4x+2}{x+2}$
$\frac{4x+2}{x+2} = \frac{35}{2(3x+1)}$
$4(2x + 1)(3x + 1) = 35x + 70$
$4(6x^2 + 5x + 1) = 35x + 70$
$24x^2 + 20x + 4 = 35x + 70$
$24x^2 - 15x - 66 = 0$
$8x^2 - 5x - 22 = 0$
$(8x + 11)(x - 2) = 0$
$x = \frac{-11}{8}$ or 2

48. $\frac{3x}{2y^4}$

49. $\frac{10b}{3}$

50. 4

51. 2×8×5×2 = 160

52. $1 - \frac{5}{7} = \frac{2}{7}\frac{2}{7} \times \frac{7}{8} = \frac{1}{4}$

53. 150÷6 = 25
$25 = 5^2$
$5^3 =$ 125 in$^3$

54. E
Try some examples.

55. $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

56. 3000×9% = □×12%
3000×9 = □×12
□ = 3000×$\frac{3}{4}$ = 2250

57. 0.7×□ = 21.63
□ = 21.63÷0.7 = $30.90

58. 40-32 = 8 (discount), the rate of discount is 8÷40 = .20 = 20%

59. 35 + 45 = 80 mph (relative speed)
60÷80 = $\frac{3}{4}$ (hr)

60. $\frac{1}{2} + \frac{1}{6} = \frac{1}{3}$
$1 \div \frac{1}{3} =$ 3 hr

61. B
$1 + 2 + 3 + \ldots + N = \frac{N(N-1)}{2}$
$2 \times \frac{N(N-1)}{2} = N(N - 1)$

62. D
3×2 = 6

63. $3^2 - 1^2 = 8$
$7^2 - 5^2 = 24$
8 : 24 = 1 : 3

64. $(x^2-4)(x^2-9) = (x+2)(x-2)(x+3)(x-3)$

65. $0.01(a+9b)(a-6b)$

66. C

67. C
$21-4x=(2x-3)^2=4x^2-12x+9$
$4x^2-8x-12=0$
$x^2-2x-3=0$
$(x - 3)(x + 1) = 0$
$x =$ 3
Note -1 is ruled out as it negates the right side, but the range of the square root function is ≥ 0.

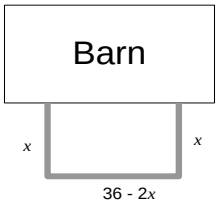
68. D
$48x-71=(1-3x)^2=9x^2-6x+1$
$9x^2-54x+72=0$
$x^2-6x+8=0$
$(x - 2)(x - 4) = 0$
No solution
Note 2 and 4 are ruled out as they negates the right side, but the range of the square root function is ≥ 0.

69. (-300, -400)

70. 25

71. A
Let the three sides be $x$, $x$, and $36 - 2x$.
The area of the pen is
$A(x) = x(36 - 2x) = 36x - 2x^2$.
The quadratic function will attain its maximum at the line of symmetry:

$\frac{36}{4} = 9$, so the greatest area is $9 \times 18 = \boxed{162 \text{ ft}^2}$.




72. C
$\frac{1}{2}r + 21 = 3h^2$
$r + 42 = 6h^2$
$h = 3$ (since 2 is too small)
$r = 54 - 42 = \boxed{12}$

73. E
(A) -72 (B) 0.013888889 (C) -1.125 (D) 0.888888889 (E) 1.125

74. C
17'5" - 5'4" = 12'1"
12'1" - 3" = 11'10"
11'10"÷2 = 5'11"
5'11" + 3" = 5'14" = 6'2"

75. B
17

76. A
24 + 36 = 60
720÷60 = 12
12×2 = $\boxed{24 \text{ rolls}}$

77. B
720÷2 = 360
360÷36 = 10 (36-shot film).
360÷24 = 15 (24-shot film).
10+15 = $\boxed{25 \text{ rolls}}$

78. E
BC = 5
AC = 13
AB = 12
12×10 = 120

79. C
Let $x$ be the length, then the width should be $\frac{3}{4}x$.
The area of the rectangle is
$\frac{3}{4}x^2 = 108$
$x^2 = 108 \times \frac{4}{3}$
$x^2 = 144$
$x$=12 (-12 is precluded.) Thus, the length is 12 m and the width is $\frac{3}{4}(12) = \boxed{9 \text{ m}}$.

80. C
Method I)
$86 \times 25 = 10 \times 92 + 10 \times 84 + 5x$
$2150 = 920 + 840 + 5x$
$x = \boxed{78}$

Method II)
Compared to the overall average, the first group has a total surplus of 6×10 = 60 pts. The second group has deficit of 2×10 = 20 pts. The third group must have a deficit of the 40 pts since 60 = 20 + 40. Therefore, each individual of the third group should have 40÷5 = 8 pts lower than the overall average, thus the average of the third group is 86 - 8 = 78.